Crop Planning in Sustainable Agriculture: Dynamic Farmland Allocation in the Presence of Crop Rotation Benefits

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Abstract

This paper examines crop planning decision in sustainable agriculture—that is, how to allocate farmland among multiple crops in each growing season when the crops have rotation benefits across growing seasons. We consider a farmer who periodically allocates the farmland between two crops in the presence of revenue uncertainty where revenue is stochastically larger and farming cost is lower when a crop is grown on rotated farmland (where the other crop was grown in the previous season). We characterize the optimal dynamic farmland allocation policy. Using a calibration based on a typical farmer growing corn and soybean in Iowa we provide rules of thumb for the effect of revenue uncertainty. In particular, we show that the farmer always benefits from a higher corn revenue volatility but benefits from a higher soybean revenue volatility only when this volatility is high; otherwise a lower soybean volatility is beneficial. We also show that growing only one crop over the entire planning horizon, as employed in industrial agriculture, leads to a considerable profit loss—that is, making crop planning based on principles of sustainable agriculture has substantial value. We propose a simple sustainable heuristic allocation policy and characterize the periodic allocation decision of this policy in closed form. Using our model calibration we show that the proposed policy not only outperforms the commonly suggested heuristic policies in the literature, but also provides a near-optimal performance.

Keywords: Farm Planning, Crop Rotation, Sustainability, Agriculture, Commodity, Uncertainty, Dynamic Programming, Corn, Soybean

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1 Introduction

Sustainable agriculture aims at growing food in an ecologically and ethically responsible manner by using practices that enhance environmental quality and natural resource base (e.g., land, soil, water) while maintaining the economic viability of farm operations. The focus on sustainable agriculture is increasing owing to a variety of factors including surging demand for food—there will be 2 billion people more to feed by 2050—and heightened economic implications of agriculture—it is the main source of income for more than a third of world's population (European Commission 2012). In this paper we study a key decision in sustainable agriculture, crop planning—that is, how should a farmer allocate the available farmland among multiple crops in each growing season?

In sustainable agriculture crop planning is made based on crops that have rotation benefits across growing seasons (USDA 2015a). When two crops have rotation benefits growing a crop on rotated farmland (where the other crop was grown in the previous season) is more profitable than growing it on non-rotated farmland (where the same crop was grown in the previous season). As highlighted by Hennessy (2006), these rotation benefits can be attributed to increasing crop revenues owing to improved soil structure and broken reproductive cycles of pests, and to decreasing farming costs owing to reduced need for fertilizers (as a result of improved soil structure) and pesticides (as a result of lower pest populations). Consider, for example, corn and soybean, the two most planted crops in the U.S.[1] Rotating these two crops is beneficial because, for instance, soybean improves the soil structure by fixing its nitrogen content, which is crucial for corn growth, and at the same time reduces the added nitrogen (fertilizer) need for corn (Livingston et al. 2015). Rotating corn with soybean also breaks the reproductive cycle of corn rootworm—the most common corn insect—and reduces the need for corn rootworm insecticide (pesticide).

Making the crop planning based on multiple crops with rotation benefits is a part of sustainable agriculture because it reduces the need for synthetic chemicals (e.g., fertilizers and pesticides), improves the soil structure, increases the biodiversity in the farm, and enhances the resilience of the farmer to adverse environmental conditions (e.g., unfavorable weather conditions, high infestation of pests and diseases) because all crops are less likely

[1] In the U.S., corn and soybean account for 55.5% of total acres harvested in 2014 (USDA 2015b) with an estimated total market value of $92 billion in the same year (USDA 2015c). Because both crops are planted within the same time period—between late March and June—they compete for the allocation of farmland.

to be affected in the same manner. It also improves the local community's diet because multiple crops are grown simultaneously. All these effects are in line with the objectives of sustainable agriculture as defined by the 1990 U.S. Farm Bill (USDA 2015a). In this paper we focus on the economic implications of crop planning in sustainable agriculture and examine how it affects farmer's profitability.

An important feature of crop planning decision is that crop revenue is uncertain in each growing season. The revenue uncertainty of each crop is driven by the uncertainty in its harvest volume and the uncertainty in its sales price at the end of the growing season. The harvest volume is uncertain owing to uncertain weather conditions and potential infestation of pests and diseases during the growing season (Kazaz and Webster 2011). The sales price is uncertain because it is typically tied to the prevailing price at the regional exchange (spot) markets (Goel and Tanrısever 2013). In practice crop revenues show considerable variability, as illustrated for corn and soybean in Figure 1.

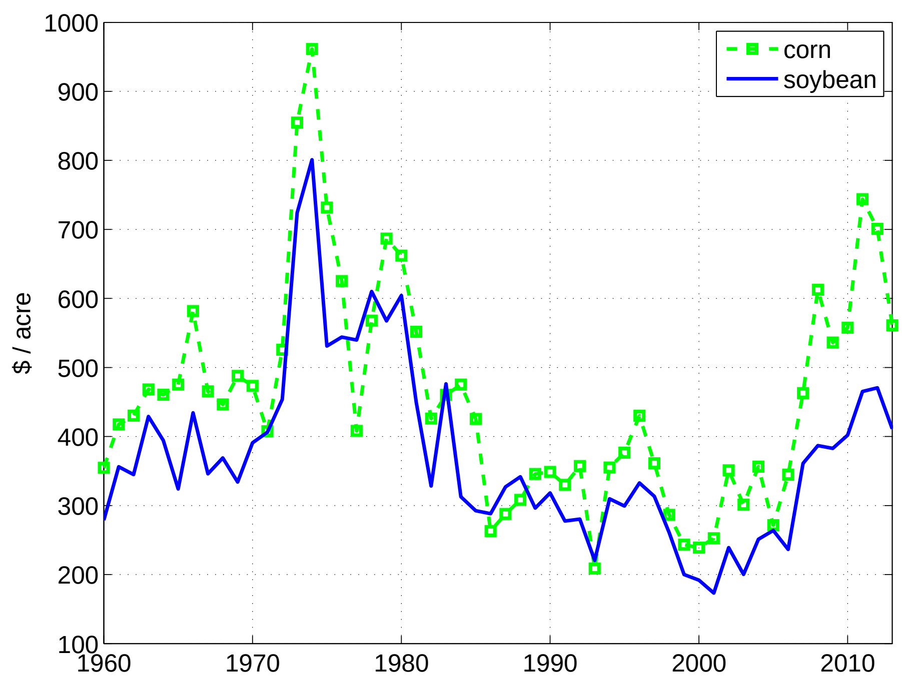

Figure 1: Annual corn and soybean revenues per acre (in $\$/acre$) in Iowa for the period 1960 to 2013 as calculated from the data reported by the U.S. Department of Agriculture.

As reviewed by Glen (1987) and more recently by Lowe and Preckel (2004), crop planning (or farmland allocation) problem has received considerable attention both in the operations management and agricultural economics literatures. The majority of papers in these literatures focuses on either single-period models where crop rotation benefits are irrelevant, or deterministic multi-period models where revenue uncertainty is absent. As highlighted by Livingston et al. (2015), the few papers which consider both revenue uncertainty and crop rotation benefits propose heuristic allocation policies and evaluate their performance using numerical experiments. In summary, there is no work that characterizes the optimal dynamic allocation policy under revenue uncertainty in the presence of crop rotation benefits. Therefore, there is also no work that examines the effect of key factors (e.g., revenue variability) on the farmer's optimal profit. In this paper we attempt to fill this void.

Toward this end, we consider a multi-period optimization problem in which a farmer decides how to allocate the available farmland between two crops in each growing season to maximize the total expected profit over a finite planning horizon. In each season (period) the allocation decision is made with respect to revenue uncertainty of each crop while considering the crop rotation benefits across seasons. We characterize the optimal dynamic farmland allocation policy and answer the following research questions.

(1) How does revenue uncertainty affect the farmer's profitability?

(2) What is the additional value of making crop planning based on multiple crops with rotation benefits, as employed in sustainable agriculture, over continuously growing only one of the crops, as employed in industrial agriculture (USDA 2015a)?

(3) How do the performance of heuristic allocation policies commonly suggested in the extant literature compare to that of the optimal policy? And, is there a simple sustainable heuristic policy that can be obtained from our analysis?

In answering these questions when analytical results are not attainable we conduct numerical experiments using realistic instances. To this end, we calibrate our model to represent a farmer growing corn and soybean in Iowa—the largest corn and soybean producing state in the U.S. based on total acreage planted and harvested in 2014 (USDA 2015b). The model calibration is based on the publicly available data from United States Department of Agriculture as complemented by the data obtained from the extant literature. Our main findings can be summarized as follows.

(1) We characterize the optimal dynamic farmland allocation policy and identify two strategies that emerge as a part of the optimal policy: *rotate*, where each crop is planted

only on the rotated farmland; and *monoculture*, where only one of the crops is planted on the entire farmland. We provide specific conditions under which each strategy is optimal.

(2) We conduct sensitivity analysis, both analytically and numerically, to investigate the effects of revenue correlation between the two crops, and revenue volatility of each crop on farmer's profitability. We show that the farmer always benefits from a lower revenue correlation but benefits from a lower revenue volatility only when this volatility is low; otherwise, a higher volatility is beneficial. Based on our model calibration we find that corn revenue volatility in practice is high so a typical farmer always benefits from a higher corn volatility. In contrast, soybean revenue volatility in practice is not as high and so the farmer may benefit from a lower soybean volatility. These results underscore the significant differences between crops based on how revenue uncertainty shapes the farmer's profitability.

(3) Using our model calibration we find that a farmer growing only one of the crops over the entire planning horizon, as employed in industrial agriculture, incurs a considerable profit loss—a minimum profit loss of 9.68% in the numerical instances considered—in comparison with a farmer using the optimal allocation policy. This result indicates that making crop planning based on principles of sustainable agriculture has substantial economic value.

(4) Based on our theoretical analysis we suggest a simple and practically implementable sustainable heuristic allocation policy where the periodic allocation decision is made based on a two-period horizon. We characterize the optimal allocation decision of this policy in closed form. Using our model calibration we find that the proposed policy not only outperforms the commonly suggested heuristic allocation policies in the literature (e.g., rotation-based policy where each crop is planted only on rotated farmland and myopic policy where the allocation decision is made based on a single-period horizon) but also provides a near-optimal performance—a maximum profit loss of 0.13% in the numerical instances considered. These results have important managerial implications. In practice farmers may feel hesitant to make the allocation decision by considering a long-term planning horizon due to complexity of such a decision. Our results demonstrate that making that allocation decision by considering a short-term horizon (specifically, two-period horizon) does not lead to a significant loss in profit, and our analysis provides a recipe for making that decision.

The remainder of this paper is organized as follows. §2 surveys the related literature and discusses the contribution of our work. §3 describes the general model and the basis for our assumptions. By focusing on a special case of our model that limits the planning horizon to two periods, §4 derives the optimal allocation policy in closed form and analytically characterizes the effect of revenue uncertainty on the farmer's profitability. §5 extends the characterization of the optimal allocation policy to the general multi-period model and provides a practical application in the context of a farmer growing corn and soybean. In particular, using a model calibration that represents a farmer in Iowa we examine the effect of revenue uncertainty on the farmer's profitability and the value of making crop planning based on principles of sustainable agriculture. We also compare the optimal allocation policy's performance with that of heuristic allocation policies. §6 concludes with a discussion of the limitations of our analysis and future research directions.

2 Literature Review

Crop planning problem has received considerable attention from the operations management and agricultural economics literatures. We refer the reader to Glen (1987), Lowe and Preckel (2004) and Ahamuda and Villalobos (2009) for a review of papers that study the crop planning problem under certainty and focus in here on papers that incorporate uncertainty. The majority of these papers considers a single-period model where crop rotation benefits are irrelevant and examines the interplay between the crop planning decision and operational features including penalties associated with cash flow variability (Collender and Zilberman 1985), government price support for crops (Chavas and Holt 1990), other government interventions (Kazaz et al. 2014), rainfall uncertainty (Maatman et al. 2002) and management of that uncertainty through irrigation planning (Huh and Lall 2013). Only a few papers in the literature consider crop rotation benefits in crop planning and study the farmland allocation problem under uncertainty in a dynamic setting. The focus of these papers is to propose heuristic allocation policies and evaluate their performance using numerical experiments (see Livingston et al. (2015) for a review). Among these papers Taylor and Burt (1984) consider a farmer's decision of whether to grow wheat or leave the farmland lie fallow in a growing season. They develop a heuristic policy and numerically analyze the policy's performance using a calibration based on a typical wheat farmer in Montana. Considering the farmland allocation between corn and soybean, Cai et al. (2013) numerically compare the performance of different heuristic allocation policies such as growing each crop only on rotated farmland and growing only one crop over the entire planning horizon.

Closest to our work, Livingston et al. (2015) examine the farmland allocation decision between corn and soybean in a multi-period framework considering the crop rotation benefits. They formulate an infinite horizon stochastic dynamic programming model where in each period the farmer chooses which one of the two crops to grow and the amount of fertilizer to use for cultivation facing uncertainties in fertilizer cost and crop revenue. They do not provide a theoretical characterization of the optimal solution and instead numerically analyze the farmer's decisions. Their main conclusion is to suggest that the farmer should implement a rotation-based heuristic allocation policy—that is, grow one of the crops in one season and rotate to the other crop in the subsequent season. In contrast to their work, we do not consider fertilizer application decision or fertilizer cost uncertainty but we extend their model to consider the possibility of growing more than one crop in the same season—a future research direction suggested in their paper. We show that consideration of that possibility is important for a farmer that employs a rotation-based heuristic allocation policy. More importantly, we characterize the optimal dynamic allocation policy based on which we propose a simple heuristic policy. Using our model calibration we show that the proposed policy outperforms the rotation-based allocation policy and provides a near-optimal performance. In addition, we offer insights that are of practical importance to the farmers on how revenue uncertainty of each crop shapes their profitability.

Our paper is also related to the growing operations management literature that examines operational decisions of supply chain agents in the agricultural sector. The majority of papers in this literature focuses on processors and studies their strategic (e.g., capacity investment) and operating (e.g., procurement and production planning) decisions. These papers consider idiosyncratic features of different agricultural industries including beef (Boyabatlı et al. 2011), citrus fruit (Kazaz and Webster 2011), cocoa (Boyabatlı 2015), corn (Goel and Tanrısever 2013), olive (Kazaz 2004), palm oil (Boyabatlı et al. 2014, Sunar and Plambeck 2015), seed (Jones et al. 2001, Burer et al. 2008), soybean (Devalkar et al. 2011) and wine (Noparumpa et al. 2015). There are also papers that focus on commoditized industries outside of the agricultural sector but their research questions are also relevant in the context of agricultural industries. For example, Chen et al. (2013) examine the processing decision of a semi-conductor manufacturer in a co-production environment where a single input gives rise to multiple outputs. That production environment is also relevant for several agricultural industries including grains and oilseeds. Another example is Plambeck and Taylor (2013) who examine process improvement investment decision of a clean-tech manufacturer that faces input and output price uncertainties. Similar investment decision is also relevant for an agri-processor (e.g., soybean crusher) who faces input (soybean) and output (soybean oil) spot price uncertainties. Dong et al. (2014) study the value of two types of operational flexibility in an oil refinery, range flexibility (the ability to process crude oil of diverse quality) and conversion flexibility (the ability to convert low-quality crude oil to high-quality crude oil). Those two types of operational flexibility are also relevant for a vegetable oil (e.g., palm oil) refinery. Paralleling the papers cited here we use methods and findings from financial engineering—for example, modeling of correlated bivariate uncertainty and its evolution over multiple periods, approximating that stochastic evolution using a lattice approach for numerical computation and insights related to how a financial option's value is affected by its volatility—and apply those in a specific problem domain—that is, crop planning in sustainable agriculture. Our analytical sensitivity results—i.e., how revenue correlation between two crops and revenue volatility of each crop affect farmer's profitability—are reminiscent of the sensitivity results in Plambeck and Taylor (2013), Boyabatlı et al. (2014) and Dong et al. (2014) who apply the insights from financial engineering in the context of a clean-tech manufacturer's process improvement decision, oilseed processor's capacity investment decision and oil refinery's technology investment decision respectively. That being said, our sensitivity results are new to the literature on crop planning. They underscore the significant differences between crops based on how revenue uncertainty shapes the farmer's profitability.

Our work also relates to the literature on sustainable operations. As highlighted by Kleindorfer et al. (2005), the main objective in this literature is to consider environmental (and natural resource) consequences of operational decisions and to help decision makers devise profitable operational practices to enhance environmental quality by, for instance, reducing greenhouse gas emissions (Plambeck 2012), converting waste into a saleable byproduct (Lee 2012) or energy (Ata et al. 2012). In the context of agricultural industries, there is a growing interest in the operations management literature that examines sustainability related issues; see Li et al. (2014) for a recent review. This literature mainly focuses on downstream supply chain (i.e., food retailers) and considers the impact of operating (e.g., procurement, processing, inventory) decisions on food waste—see, for instance, Lee and Tongarlak (2014). Our paper's focus is on the upstream supply chain (i.e., farmers) and we study sustainable way of making crop planning. As highlighted on page 211 of National

Research Council (2010), there is vast amount of research in agricultural economics that examines the environmental impact of farming practices such as rotating crops—that is, environmental benefits of making crop planning based on crops with rotation benefits is well-established—yet research that examines the economic impact of those farming practices is limited. Based on this important lacuna in the literature we focus on the economic implications of crop planning decision and examine how it affects farmer's profitability. Using a model calibration we demonstrate that making crop planning based on the principles of sustainable agriculture has substantial economic value.

3 Model Description and Assumptions

The following mathematical representation is used throughout the text: A realization of the random variable $\tilde{y}$ is denoted by y. The expectation operator is denoted by $\mathbb{E}$. Bold face letters represent row vectors of the required size. We use $(u)^+ = \max(u, 0)$ and $-j = S \setminus j$ for $j \in S$. All the proofs are relegated to the Appendix.

We consider a farmer who allocates a farmland between two crops in each growing season to maximize the expected total profit over a finite number of growing seasons. The farmland is fixed throughout the planning horizon which we normalize to one acre without loss of generality. Though our model is generic, for the concreteness of the exposition we label the two crop choices available for the farmer as corn (c) and soybean (s). We use superscript c (s) to denote the corn (soybean) related parameters.

In each growing season (time period) t, the farmer allocates a proportion $\alpha_t \in [0, 1]$ of the farmland to corn with the remaining $1 - \alpha_t$ proportion allocated to soybean. The allocation decision α_t is made with respect to revenue uncertainty of each crop. Let $\tilde{r}_t^c$ and $\tilde{r}_t^s$ denote the uncertain corn and soybean revenue per acre in period t, respectively. We assume that $\tilde{\boldsymbol{r}}_{\boldsymbol{t}} = (\tilde{r}_t^c, \tilde{r}_t^s)$ follow correlated stochastic processes with Markovian property; that is, the current revenue realizations are sufficient to characterize the distribution of the future revenues. We make further assumptions about these stochastic processes later in §4.2 to study the effect of revenue uncertainty.

A key feature of the farmland allocation decision is the crop rotation benefits across growing seasons. In particular, profit from growing a crop on rotated farmland, where the other crop was grown in the previous season, is (stochastically) larger than the profit from growing it on non-rotated farmland, where the same crop was grown in the previous season.

As discussed in §1, the crop rotation benefits are attributed to *i*) increasing crop revenues and *ii*) decreasing farming costs. To capture the revenue-enhancing crop rotation benefits, we assume that the uncertain revenue per acre of crop $j \in \{c, s\}$ grown on rotated farmland in period t is $(1 + b^j)\tilde{r}_t^j$ where $b^j \geq 0$. The uncertain revenue per acre of the same crop grown on non-rotated farmland is $\tilde{r}_t^j$. To capture the cost-reducing crop rotation benefits, we assume that the unit farming cost of crop j is f^j if it is grown on non-rotated farmland, and $(1 - \gamma^j)f^j$ for $\gamma^j \geq 0$ if it is grown on rotated farmland. When $b^j = 0$ and $\gamma^j = 0$, there is no rotation benefit for crop $j \in \{c, s\}$. Our model can be extended to include period-dependent farming cost f^j, and crop rotation parameters b^j and γ^j. Because it does not affect the structural analysis, for brevity, we assume that they are fixed throughout the planning horizon. We also assume that crop rotation benefits carry through for only one period and the allocation decision α_t in period t is impacted by only the allocation in period $t - 1$ and not the earlier periods. This is a reasonable assumption for corn-soybean rotation as documented in Hennessy (2006).

We formulate the farmer's problem as a finite horizon stochastic dynamic program. In each period $t \in [1, T]$, the sequence of events is as follows: (i) at the beginning of period t, the farmer observes the corn allocation α_{t-1}, and corn and soybean revenues $\boldsymbol{r_{t-1}} = (r_{t-1}^c, r_{t-1}^s)$ from period $t - 1$ and chooses the corn allocation α_t; (ii) at the end of period t, the corn and soybean revenues $\boldsymbol{\tilde{r}_t} = (\tilde{r}_t^c, \tilde{r}_t^s)$ are realized and the farmer collects the revenues from the crop sales. The farmer's immediate payoff in period $t \in [1, T]$ is given by

$$\begin{aligned} L(\alpha_t \mid \alpha_{t-1}, \boldsymbol{r_{t-1}}) \doteq & - \quad (\alpha_t - \gamma^c \min(\alpha_t, 1 - \alpha_{t-1}))\, f^c - (1 - \alpha_t - \gamma^s \min(1 - \alpha_t, \alpha_{t-1}))\, f^s \quad (1) \\ & + \quad \mathbb{E}_t \left[(\alpha_t + b^c \min(\alpha_t, 1 - \alpha_{t-1}))\, \tilde{r}_t^c + (1 - \alpha_t + b^s \min(1 - \alpha_t, \alpha_{t-1}))\, \tilde{r}_t^s \right], \end{aligned}$$

where $\mathbb{E}_t[.]$ denotes the expectation operator conditional on the available information at time t, i.e., $\mathbb{E}_t[.] = \mathbb{E}[.|\boldsymbol{r_{t-1}}]$. In (1), the first line corresponds to the total farming cost and the second line corresponds to the expected revenues from crop sales in period t. When the farmer decides to allocate α_t proportion of the farmland to corn, to leverage crop rotation benefits, the farmer starts planting corn from the rotated farmland (where soybean was grown in the previous period) which is $1 - \alpha_{t-1}$ proportion of the farmland. Therefore, rotation benefits for corn plantation are only relevant for $\min(\alpha_t, 1 - \alpha_{t-1})$ proportion of the farmland which yields the revenue $(1+b^c)\tilde{r}_t^c$ with the farming cost $(1-\gamma^c)f^c$. Similarly, rotation benefits for soybean plantation are only relevant for $\min(1 - \alpha_t, \alpha_{t-1})$ proportion of the farmland which yields the revenue $(1 + b^s)\tilde{r}_t^s$ with the farming cost $(1 - \gamma^s)f^s$.

Let $V_t(\alpha_{t-1}, \boldsymbol{r_{t-1}})$ for $t \in [1, T]$ denote the optimal value function from period t onwards given α_{t-1} and $\boldsymbol{r_{t-1}}$, which satisfies

$$V_t(\alpha_{t-1}, \boldsymbol{r_{t-1}}) = \max_{0 \le \alpha_t \le 1} \Big\{ L(\alpha_t \mid \alpha_{t-1}, \boldsymbol{r_{t-1}}) + \mathbb{E}_t[V_{t+1}(\alpha_t, \boldsymbol{\tilde{r}_t})] \Big\}, \tag{2}$$

with a boundary condition $V_{T+1}(.) = 0$. The farmer's optimal total expected profit over the entire planning horizon is given by $V_1(\alpha_0, \boldsymbol{r_0})$, where α_0 and $\boldsymbol{r_0}$ denote the observed corn allocation and crop revenues at the beginning of the planning horizon, respectively.

4 Two-Period Model

In this section we limit the planning horizon to two periods, i.e., $T = 2$, and characterize the optimal allocation policy (§4.1) and conduct sensitivity analysis to examine the effect of revenue uncertainty on the farmer's profitability (§4.2). Focusing on a two-period model enables us to analytically characterize the sensitivity results and to provide a closed-form solution for the optimal allocation decision which we use as a heuristic policy for a general T-period problem. We examine how the optimal policy characterization and the sensitivity results extend to the T-period problem in §5.

4.1 Optimal Allocation Policy

We solve the farmer's problem using backward induction. In period 1, the farmer allocates α_1 proportion of the farmland to corn and observes crop revenues $\boldsymbol{r_1} = (r_1^c, r_1^s)$. In period 2, the farmer chooses the corn allocation α_2. The farmer's optimization problem in this period follows from (1) by substituting $t = 2$ and using the boundary condition $V_3(.) = 0$.

Proposition 1 *In period 2, the optimal corn allocation α_2^* is given by*

$$\alpha_2^* = \begin{cases} 0 & \textit{if } -(1-\gamma^c)f^c + \mathbb{E}_2\left[(1+b^c)\tilde{r}_2^c\right] \le -f^s + \mathbb{E}_2\left[\tilde{r}_2^s\right], \\ 1 & \textit{if } -f^c + \mathbb{E}_2\left[\tilde{r}_2^c\right] \ge -(1-\gamma^s)f^s + \mathbb{E}_2\left[(1+b^s)\tilde{r}_2^s\right], \\ 1-\alpha_1 & \textit{otherwise.} \end{cases} \tag{3}$$

The optimal expected profit in this period is $V_2(\alpha_1, \boldsymbol{r_1}) = \alpha_1 K_2^c(\boldsymbol{r_1}) + (1-\alpha_1) K_2^s(\boldsymbol{r_1})$ where

$$\begin{aligned} K_2^c(\boldsymbol{r_1}) &\doteq \max\left\{-f^c + \mathbb{E}_2\left[\tilde{r}_2^c\right], -(1-\gamma^s)f^s + \mathbb{E}_2\left[(1+b^s)\tilde{r}_2^s\right]\right\}, \\ K_2^s(\boldsymbol{r_1}) &\doteq \max\left\{-(1-\gamma^c)f^c + \mathbb{E}_2\left[(1+b^c)\tilde{r}_2^c\right], -f^s + \mathbb{E}_2\left[\tilde{r}_2^s\right]\right\}. \end{aligned} \tag{4}$$

To reap the crop rotation benefits, the farmer has an incentive to grow each crop on rotated farmland. However, it can be profitable to break the rotation and grow *additional* crop

$j \in \{c, s\}$ on non-rotated farmland if this crop's profit is expected to be *sufficiently* larger than the other crop's profit. The optimal allocation decision in (3) follows this intuition. In particular, if the expected marginal profit of growing corn on non-rotated farmland is larger than the expected marginal profit of growing soybean on rotated farmland then the farmer only grows corn, i.e., $\alpha_2^* = 1$. Similarly, if the expected marginal profit of growing soybean on non-rotated farmland is larger than the expected marginal profit of growing corn on rotated farmland then the farmer only grows soybean, i.e., $\alpha_2^* = 0$. Otherwise, the farmer optimally grows each crop on rotated farmland, i.e., $\alpha_2^* = 1 - \alpha_1$. The optimal expected profit in period 2 also follows an intuitive structure: it is characterized by the product of the proportion of farmland allocated to crop $j \in \{c, s\}$ in period 1—α_1 for corn and $1 - \alpha_1$ for soybean—and its expected marginal profit in period 2—$K_2^j(\boldsymbol{r_1})$ in (4). Here, $K_2^j(\boldsymbol{r_1})$ is given by the maximum profit from two options available to the farmer, growing crop j—which does not involve rotation benefits as it is grown on non-rotated farmland—and growing crop $-j$—which involves rotation benefits as it is grown on rotated farmland.

We now proceed to characterize the optimal allocation decision in period 1. The farmer's optimization problem in this period follows from (1) by substituting $t = 1$ and using the characterization of $V_2(\alpha_1, \boldsymbol{r_1})$ as given by Proposition 1.

Proposition 2 *In period 1, the optimal corn allocation α_1^* is given by*

$$\alpha_1^* = \begin{cases} 0 & \text{if } -(1-\gamma^c)f^c + \mathbb{E}_1\left[(1+b^c)\tilde{r}_1^c + K_2^c(\tilde{\boldsymbol{r}}_1)\right] \le -f^s + \mathbb{E}_1\left[\tilde{r}_1^s + K_2^s(\tilde{\boldsymbol{r}}_1)\right], \\ 1 & \text{if } -f^c + \mathbb{E}_1\left[\tilde{r}_1^c + K_2^c(\tilde{\boldsymbol{r}}_1)\right] \ge -(1-\gamma^s)f^s + \mathbb{E}_1\left[(1+b^s)\tilde{r}_1^s + K_2^s(\tilde{\boldsymbol{r}}_2)\right], \\ 1-\alpha_0 & \text{otherwise,} \end{cases} \quad (5)$$

where $K_2^j(\boldsymbol{r_1})$ for $j \in \{c, s\}$ is as given in (4). The optimal total expected profit over the planning horizon is $V_1(\alpha_0, \boldsymbol{r_0}) = \alpha_0 K_1^c(\boldsymbol{r_0}) + (1-\alpha_0)K_1^s(\boldsymbol{r_0})$ where

$$K_1^c(\boldsymbol{r_0}) \doteq \max\left\{-f^c + \mathbb{E}_1\left[\tilde{r}_1^c + K_2^c(\tilde{\boldsymbol{r}}_1)\right], -(1-\gamma^s)f^s + \mathbb{E}_1\left[(1+b^s)\tilde{r}_1^s + K_2^s(\tilde{\boldsymbol{r}}_1)\right]\right\}, \quad (6)$$

$$K_1^s(\boldsymbol{r_0}) \doteq \max\left\{-(1-\gamma^c)f^c + \mathbb{E}_1\left[(1+b^c)\tilde{r}_1^c + K_2^c(\tilde{\boldsymbol{r}}_1)\right], -f^s + \mathbb{E}_1\left[\tilde{r}_1^s + K_2^s(\tilde{\boldsymbol{r}}_1)\right]\right\}.$$

The optimal allocation decision in (5) follows the same structure as Proposition 1: if the expected marginal profit of growing corn (soybean) on non-rotated farmland is larger than the expected marginal profit of growing soybean (corn) on rotated farmland then the farmer only grows corn (soybean), i.e., $\alpha_1^* = 1$ ($\alpha_1^* = 0$); otherwise, the farmer optimally grows each crop on rotated farmland, i.e., $\alpha_1^* = 1 - \alpha_0$. The only difference from Proposition 1 is that the expected marginal profit expressions not only include the profit in period 1 but also the profit in period 2 which is captured by $K_2^j(\boldsymbol{r_1})$, $j \in \{c, s\}$. For example, expected marginal profit of growing corn on rotated farmland is given by the sum of the expected corn profit in period 1, i.e., $-(1-\gamma^c)f^c + \mathbb{E}_1\left[(1+b^c)\tilde{r}_1^c\right]$, and the expected marginal profit in period 2 obtained from the farmland where corn was grown in period 1, i.e., $\mathbb{E}_1\left[K_2^c(\boldsymbol{\tilde{r}_1})\right]$.

The characterization of the optimal total expected profit over the entire planning horizon also follows the same structure as Proposition 1. In particular, this profit is given by the product of the proportion of farmland allocated to crop $j \in \{c, s\}$ at the beginning of the planning horizon (period 0)—α_0 for corn and $1-\alpha_0$ for soybean—and its expected marginal profit over the entire planning horizon—$K_1^j(\boldsymbol{r_0})$ in (6). Here, $K_1^j(\boldsymbol{r_0})$ is given by the maximum profit from two options available to the farmer: (i) growing crop j in period 1 and optimally using the farmland in period 2 (which yields the expected marginal profit $\mathbb{E}_1\left[K_2^j(\boldsymbol{\tilde{r}_1})\right]$) and (ii) growing crop $-j$ in period 1 and optimally using the farmland in period 2 (which yields the expected marginal profit $\mathbb{E}_1\left[K_2^{-j}(\boldsymbol{\tilde{r}_1})\right]$).

Proposition 2 identifies two strategies that emerge as a part of the optimal allocation policy: *rotate*, where each crop is only grown on rotated farmland, and *monoculture*, where only of the crops is grown on the entire farmland. This proposition provides specific conditions under which each strategy is optimal. We will later show in §5 that this optimal policy structure extends to a more general T-period model.

4.2 Effect of Revenue Uncertainty

In this section we examine the effect of revenue uncertainty on the farmer's profitability. To this end, we impose additional structure on our model of the revenue processes. In particular, paralleling Boyabatlı et al. (2014), we use a single-factor bi-variate mean-reverting process to describe the evolution of the corn and soybean revenues. Specifically, corn and soybean revenues at time τ, $\boldsymbol{r_\tau} = (r_\tau^c, r_\tau^s)$, are modeled as

$$
\begin{aligned}
dr_\tau^c &= \kappa^c(\xi^c - r_\tau^c)d_\tau + \sigma^c d\tilde{W}_\tau^c, \qquad (7)\\
dr_\tau^s &= \kappa^s(\xi^s - r_\tau^s)d_\tau + \sigma^s d\tilde{W}_\tau^s,
\end{aligned}
$$

where $\kappa^j > 0$ is the mean-reversion parameter, ξ^j is the long-term revenue level and σ^j is the volatility for $j \in \{c, s\}$, whereas $(d\tilde{W}_\tau^c, d\tilde{W}_\tau^s)$ denotes the increment of a standard bi-variate Brownian motion with correlation ρ. We assume $\rho > 0$ throughout our analysis. This is a reasonable assumption for corn and soybean as we empirically demonstrate in §5.2.1. We discretize the revenue model in (7) and assume that time τ corresponds to period t. This revenue model implies that at period $\hat{t}$ with realized revenues $\boldsymbol{r_{\hat{t}}} = (r^c_{\hat{t}}, r^s_{\hat{t}})$, the revenues $\tilde{\boldsymbol{r}}_{\boldsymbol{t}} = (\tilde{r}^c_t, \tilde{r}^s_t)$ at a future period $t > \hat{t}$ follow a bi-variate Normal distribution with

$$\begin{aligned} \mathbb{E}[\tilde{r}^j_t | \boldsymbol{r_{\hat{t}}}] &= e^{-\kappa^j(t-\hat{t})} r^j_{\hat{t}} + \left(1 - e^{-\kappa^j(t-\hat{t})}\right) \xi^j, \\ \text{VAR}[\tilde{r}^j_t | \boldsymbol{r_{\hat{t}}}] &= \frac{1 - e^{-2\kappa^j(t-\hat{t})}}{2\kappa^j} (\sigma^j)^2, \\ \text{COV}[\tilde{r}^c_t, \tilde{r}^s_t | \boldsymbol{r_{\hat{t}}}] &= \frac{1 - e^{-(\kappa^c+\kappa^s)(t-\hat{t})}}{\kappa^c + \kappa^s} \rho \sigma^c \sigma^s. \end{aligned} \tag{8}$$

We conduct sensitivity analysis to study how revenue correlation ρ and revenue volatility σ^j of crop $j \in \{c, s\}$ impact the optimal total expected profit over the planning horizon $V_1(\alpha_0, \boldsymbol{r_0})$. To this end, we first provide a closed-form characterization of $V_1(\alpha_0, \boldsymbol{r_0})$.

Proposition 3 *Under the bi-variate normal distribution specified in (8), $V_1(\alpha_0, \boldsymbol{r_0})$ in Proposition 2 can be characterized in closed form using $K^j_2(\boldsymbol{r_1}) = \max\{\underline{u} r^j_1 + \underline{v}, \overline{u} r^{-j}_1 + \overline{v}\}$ for $j \in \{c, s\}$ where $\underline{u} = e^{-\kappa^j}$, $\underline{v} = \left(1 - e^{-\kappa^j}\right) \xi^j - f^j$, $\overline{u} = (1 + b^{-j}) e^{-\kappa^{-j}}$, $\overline{v} = (1 + b^{-j}) \left(1 - e^{-\kappa^{-j}}\right) \xi^{-j} - (1 - \gamma^{-j}) f^{-j}$ and the following identity*

$$\begin{aligned} \mathbb{E}_1[\max\{\underline{u}\tilde{r}^j_1 + \underline{v}, \overline{u}\tilde{r}^{-j}_1 + \overline{v}\}] &= (\underline{u}\mu^j_1 + \underline{v}) \Phi\left(\frac{\underline{u}\mu^j_1 + \underline{v} - \overline{u}\mu^{-j}_t - \overline{v}}{\lambda}\right) \\ &+ (\overline{u}\mu^{-j}_1 + \overline{v}) \Phi\left(\frac{\overline{u}\mu^{-j}_1 + \overline{v} - \underline{u}\mu^j_1 - \underline{v}}{\lambda}\right) + \lambda \phi\left(\frac{\overline{u}\mu^{-j}_1 + \overline{v} - \underline{u}\mu^j_1 - \underline{v}}{\lambda}\right), \end{aligned} \tag{9}$$

where $\Phi(.)$ and $\phi(.)$ denote the cumulative distribution function and the probability density function of the standard normal distribution respectively, $\lambda \doteq \sqrt{\underline{u}^2\, VAR^j_1 + \overline{u}^2\, VAR^{-j}_1 - 2\underline{u}\overline{u} COV_1}$, and $\mu^j_1 \doteq \mathbb{E}[\tilde{r}^j_1 | \boldsymbol{r_0}]$, $VAR^j_1 \doteq VAR[\tilde{r}^j_1 | \boldsymbol{r_0}]$, and $COV_1 \doteq COV[\tilde{r}^c_1, \tilde{r}^s_1 | \boldsymbol{r_0}]$ follow from (8) with $t = 1$ and $\hat{t} = 0$.

The key observation from Propositions 2 and 3 is that revenue correlation and revenue volatility of each crop affect the farmer's profitability through their impacts on the expected marginal profit from optimally using the farmland in period 2, i.e., $\mathbb{E}_1\left[K^j_2(\tilde{\boldsymbol{r}}_{\boldsymbol{1}})\right]$ for $j \in \{c, s\}$. As discussed in §4.1, this expected marginal profit is given by the maximum profit from two options available to the farmer, growing corn and growing soybean. We will use this observation to explain our sensitivity results as discussed next.

Proposition 4 (Revenue correlation ρ) $\frac{\partial V_1(\alpha_0, \boldsymbol{r_0})}{\partial \rho} < 0$.

With a higher ρ, there will be a higher likelihood that when the profit from growing one of the crops is low, the profit from growing the other crop will be low. Therefore, as ρ

increases the maximum profit from these two options, and thus the optimal expected profit over the planning horizon decreases.

Proposition 5 (Revenue volatility σ^j) *For $j \in \{c, s\}$, there exists a unique $\overline{\sigma}^j$ such that $\frac{\partial V_1(\alpha_0, \boldsymbol{r_0})}{\partial \sigma^j} \leq 0$ if $\sigma^j < \overline{\sigma}^j$; and $\frac{\partial V_1(\alpha_0, \boldsymbol{r_0})}{\partial \sigma^j} \geq 0$ if $\sigma^j > \overline{\sigma}^j$.*

With a higher σ^j, there will be a higher likelihood of observing low and high profit from crop j. When σ^j is small and increases, while a higher likelihood of observing low profit from crop j decreases the maximum profit from two options, a higher likelihood of observing high profit from crop j does not increase the maximum profit from two options. The latter argument follows because the profit from the other crop, which is also high due to the positive correlation, exceeds the profit from crop j and determines the maximum profit from two options. Therefore, when σ^j is small, a higher σ^j decreases the optimal expected profit over the planning horizon. This result is reversed when σ^j is large. In that case, as σ^j increases while a higher likelihood of observing high profit from crop j increases the maximum profit from two options, a higher likelihood of observing low profit from crop j is less consequential. The latter argument follows because the profit from the other crop, which is also low due to the positive correlation, exceeds the profit from crop j and determines the maximum profit from two options.

In summary, our sensitivity results reveal that farmer always benefits from a lower revenue correlation between two crops but benefits from a lower revenue volatility of each crop only when this volatility is low; otherwise, a higher volatility is beneficial.

5 T-Period Model and Its Application in Crop Planning with Corn and Soybean

In this section, we extend our analysis in §4 that focuses on a two-period problem to a general T-period problem. In §5.1, we demonstrate that the optimal allocation policy as characterized in §4.1 continues to hold for the T-period problem. In §5.2, we conduct numerical experiments by calibrating our model parameters to represent a typical farmer growing corn and soybean.

5.1 Optimal Allocation Policy

We now solve for the farmer's optimization problem stated in (1) and characterize the optimal allocation decision and the optimal value function in period $t \in [1, T]$.

Proposition 6 *In period $t \in [1, T]$, the optimal corn allocation α_t^* is given by*

$$\alpha_t^* = \begin{cases} 0 & \text{if } -(1-\gamma^c)f^c + \mathbb{E}_t\left[(1+b^c)\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)\right] \le -f^s + \mathbb{E}_t\left[\tilde{r}_t^s + K_{t+1}^s(\tilde{\boldsymbol{r}}_t)\right], \\ 1 & \text{if } -f^c + \mathbb{E}_t\left[\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)\right] \ge -(1-\gamma^s)f^s + \mathbb{E}_t\left[(1+b^s)\tilde{r}_t^s K_{t+1}^s(\tilde{\boldsymbol{r}}_t)\right], \\ 1-\alpha_{t-1} & \text{otherwise,} \end{cases} \quad (10)$$

where

$$\begin{aligned} K_t^c(\boldsymbol{r_{t-1}}) &\doteq \max\left\{-f^c + \mathbb{E}_t\left[\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)\right], -(1-\gamma^s)f^s + \mathbb{E}_t\left[(1+b^s)\tilde{r}_t^s + K_{t+1}^s(\tilde{\boldsymbol{r}}_t)\right]\right\}, \quad (11) \\ K_t^s(\boldsymbol{r_{t-1}}) &\doteq \max\left\{-(1-\gamma^c)f^c + \mathbb{E}_t\left[(1+b^c)\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)\right], -f^s + \mathbb{E}_t\left[\tilde{r}_t^s + K_{t+1}^s(\tilde{\boldsymbol{r}}_t)\right]\right\}. \end{aligned}$$

with $K_{T+1}^j(\boldsymbol{r_T}) = 0$. The optimal value function from period t onwards is given by

$$V_t(\alpha_{t-1}, \boldsymbol{r_{t-1}}) = \alpha_{t-1}K_t^c(\boldsymbol{r_{t-1}}) + (1-\alpha_{t-1})K_t^s(\boldsymbol{r_{t-1}}). \quad (12)$$

The characterizations of the optimal allocation decision and the optimal value function follow the same structure as the characterizations in Proposition 2 except for one modification: the recursive operators $K_1^j(\boldsymbol{r_0})$ for $j \in \{c, s\}$, as defined in (6), are generalized to suit the T-period model. In particular, $K_t^j(\boldsymbol{r_{t-1}})$ denotes the expected marginal profit of farmland in the remaining planning horizon (from period t onwards) where crop j was grown in period $t-1$. It is given by the maximum profit from two options available to the farmer: (i) growing crop j in period t and optimally using the farmland in the remaining periods (which yields the expected marginal profit $\mathbb{E}_t\left[K_{t+1}^j(\tilde{\boldsymbol{r}}_t)\right]$) and (ii) growing crop $-j$ in period t and optimally using the farmland in the remaining periods (which yields the expected marginal profit $\mathbb{E}_t\left[K_{t+1}^{-j}(\tilde{\boldsymbol{r}}_t)\right]$). It is easy to establish that Proposition 6 is identical to Proposition 2 when $t = T-1$ and it is identical to Proposition 1 when $t = T$ if the planning horizon is limited to two periods, i.e., $T = 2$.

Unlike the two-period problem, the optimal policy and the optimal value function for a general T-period problem cannot be characterized in closed form under the specific revenue processes defined in (7) because $K_t^j(\boldsymbol{r_{t-1}})$ in (11) cannot be characterized in closed form. Therefore, we resort to numerical experiments for further analysis.

5.2 Application in Crop Planning with Corn and Soybean

In this section we provide a practical application in the context of a farmer growing corn and soybean and calibrate our model parameters to represent a typical farmer in Iowa, the largest corn and soybean producing state in the U.S. based on total acreage planted and harvested in 2014 (USDA 2015b). We describe the data and calibration used for our numerical experiments in §5.2.1. Using these experiments we examine the effect of revenue uncertainty on the farmer's profitability (§5.2.2), the value of making crop planning based on principles of sustainable agriculture (§5.2.3) and the performance of a variety of heuristic allocation policies in comparison with the optimal policy (§5.2.4).

5.2.1 Data and Model Calibration for Numerical Experiments

Our numerical experiments use publicly available data from United States Department of Agriculture (USDA hereafter) complemented by data reported in other academic studies.

Calibration for Revenue Process Parameters. The revenue per acre of each crop (\$/acre) is determined by the product of its yield (bushel/acre) and sales price (\$/bushel). Because a time period in our model corresponds to a year and because corn and soybean are mostly harvested in October in Iowa, we use the annual yield data and October price data for each crop between 1960 and 2013 as reported by USDA for farmlands in Iowa. To obtain the real values for prices, we adjust the (nominal) data based on the consumer price index (available at United States Department of Labor) with a base year of 2000. For each year, we multiply the price and yield to obtain the revenue per acre of each crop as plotted in Figure 1. The obtained revenue data is a weighted average of the revenue from rotated and non-rotated farmlands. In particular, the observed revenue corresponds to $r_t^j[(1+b^j)\times\varphi_t^j+(1-\varphi_t^j)]$ where φ_t^j is the fraction of rotated farmland in year t of crop $j\in\{c,s\}$. To calibrate the revenue process parameters, we need to adjust the data for crop rotation benefits and obtain the (raw) revenue observation r_t^j. To do so, we use the values of φ_t^c and φ_t^s reported in Livingston et al. (2015) for Iowa in the years 1981-1982, 1986-1987, 1991-1992, and 1996-2007. For missing years, we use the average of the data available and obtain $\varphi_t^c=0.77$ and $\varphi_t^s=0.93$. To obtain the values of b^c and b^s we refer to Cai et al. (2013) who report the yield of corn and soybean on non-rotated farmland as 92.2% and 85.5% of the yield on rotated farmland, respectively. Based on these values we obtain $\hat{b}^c=1/(1-0.078)-1\cong 0.08$ and $\hat{b}^s=1/(1-0.145)-1\cong 0.17$. Using φ_t^j and $\hat{b}^j$, we calculate the revenue observations r_t^j from the data plotted in Figure 1.

To describe the evolution of corn and soybean revenues, we continue to use the single-factor bi-varite mean-reverting process as specified in (7) of §4.2. According to this specification, the yearly revenues evolve as (see Boyabatlı et al. 2014)

$$\begin{aligned}
\tilde{r}_t^c &= e^{-\kappa^c} r_{t-1}^c + (1-e^{-\kappa^c})\xi^c + \sigma^c \sqrt{\frac{1-e^{-2\kappa^c}}{2\kappa^c}}\tilde{z}^c; \\
\tilde{r}_t^s &= e^{-\kappa^s} r_{t-1}^z + (1-e^{-\kappa^z})\xi^s + \sigma^s \sqrt{\frac{1-e^{-2\kappa^s}}{2\kappa^s}}\tilde{z}^s,
\end{aligned} \tag{13}$$

where $(\tilde{z}^c, \tilde{z}^s)$ follows a standard bi-variate Normal distribution with correlation ρ. The equations in (13) are a system of simultaneous equations of $(\tilde{r}_t^c, \tilde{r}_t^s)$ on (r_{t-1}^c, r_{t-1}^s) with the form $\tilde{r}_t^j = \beta^j r_{t-1}^j + \eta^j + \tilde{\epsilon}^j$ for $j \in \{c, s\}$. Because the error terms $(\tilde{\epsilon}^c, \tilde{\epsilon}^s)$ are correlated, we use the Seemingly Unrelated Regression (henceforth SUR; see Zellner 1962) to estimate β^j, η^j and the covariance matrix of $(\tilde{\epsilon}^c, \tilde{\epsilon}^s)$. Based on these estimates, using (13), we obtain $\hat{\kappa}^c = 0.33$, $\hat{\xi}^c = 439.07$, $\hat{\sigma}^c = 108.22$, $\hat{\kappa}^s = 0.35$, $\hat{\xi}^s = 328.64$, $\hat{\sigma}^s = 79.69$, and $\hat{\rho} = 0.73$. The root mean-squared errors and the mean absolute percentage errors between the observed and the estimated revenues, the adjusted R^2 of the individual regression equations, and the (system-wide) R^2 of the SUR are given by Table 1.

Goodness of Fit

	Corn (c)	Soybean (s)
Root Mean Squared Error	92.4976	67.6455
Mean Absolute Percentage Error	17%	16.45%
Adjusted R^2	60.89%	62.66%
System-wide R^2	50.66%	

Table 1: Goodness of fit test result of the Seemingly Unrelated Regression estimation

Calibration for other operational parameters. We calibrate the (variable) farming cost f^j and the cost-reducing crop rotation benefit γ^j for crop $j \in \{c, s\}$ using the data presented in Iowa State University extension and outreach report[2]. Similar to this report, we assume that the variable farming cost is characterized by the cost of seeds, chemicals (fertilizers and herbicides), and the variable cost of machinery. The report presents each of these cost components between 1994 and 2013 for a typical farmer in Iowa that grows corn following corn, corn following soybean, and soybean following corn. Because there is no cost-reducing rotation benefit of soybean reported in the literature, we assume the variable cost for the farmer growing soybean after soybean (which is not reported in our data source) is the same as the variable cost for the farmer growing soybean following corn, i.e., $\hat{\gamma}^s = 0$.

[2]http://www.extension.iastate.edu/agdm/cdcostsreturns.html

To obtain the real values for the variable cost, we adjust the (nominal) data based on the consumer price index with a base year of 2000. To estimate the farming cost f^c and f^s, we take the average of the cost data for corn following corn and soybean following soybean, and obtain $\hat{f}^c = \$251.61$/acre and $\hat{f}^s = \$122.15$/acre. To estimate the cost-reducing crop rotation benefit for corn, we compare the average cost for corn following corn ($\$251.61/acre$) and corn following soybean ($\$225.68/acre$), and obtain $\hat{\gamma}^c = 1 - 225.68/251.61 = 0.103 \cong 0.10$. For the initial corn allocation α_0, we use the total farmland in Iowa where corn (13,600,000 acres) and soybean (9,950,000 acres) were grown in 2014 (USDA 2015b). We estimate α_0 by using the fraction of the total farmland allocated to corn and obtain $\hat{\alpha}_0 = 0.58$.

Numerical Computation. For numerical computation, we follow the standard procedure in the literature and discretize the continuous revenue process in (7) to a lattice. In particular, we represent the stochastic evolution of corn and soybean revenues as a two-dimensional trinomial recombinant lattice based on the method presented in Tseng and Lin (2007; see §3.2).[3] In this lattice each period t is discretized into δ time-steps which we set $\delta = 12$, i.e., each time-step corresponds to a month in practice. Each lattice node (in a given time-step) can transit to 3×3 nodes (in the subsequent time-step) that are defined by the jump sizes with a particular transition probability. The jump sizes and the transition probabilities are computed based on the formulas given in Tseng and Lin (2007). The distribution of corn and soybean revenues at period t is given by a discrete distribution represented by a set of lattice nodes at period t with a value and a probability for each node. As discussed in Tseng and Lin (2007), this discrete distribution asymptotically converges to the exact continuous distribution when $\delta \to \infty$. To evaluate the optimal policy and the optimal value function in period $t \in [1, T]$, as given by Proposition 6, we compute recursively $K_t^j(\mathbf{r_{t-1}})$ for $j \in \{c, s\}$ in (11) for any given $\mathbf{r_{t-1}}$ on the lattice.

Baseline Scenario. We obtain the optimal total expected profit over the planning horizon for the baseline scenario with a planning horizon of 10 years ($\hat{T} = 10$) as \$2543.0/acre, i.e., an annual value of \$254.30/acre (with a base year of 2000), which is \$352.41/acre in 2015 dollars. This is comparable with the values reported by Economic Research Services (2015):

[3] This method extends the commonly used discretization method of Jaillet et al. (2004) that is defined for a univariate continuous process to a bi-variate continuous process.

an average farmer in the U.S. growing corn earns \$364.91/acre and \$246.26/acre in 2013 and 2014, respectively, which is \$373.81/acre and \$248.24/acre, respectively, in 2015 dollars; an average farmer in the U.S. planting soybean earns \$359.92/acre and \$317.66/acre in 2013 and 2014, respectively, which is \$368.70/acre and \$320.21/acre, respectively, in 2015 dollars.

5.2.2 Effect of Revenue Uncertainty

In this section, we numerically examine the effects of revenue correlation between the two crops and revenue volatility of each crop on the farmer's optimal total expected profit over the planning horizon. We compare our numerical results with the analytical results obtained from the two-period problem in §4.2.

Figure 2 illustrates the effect of revenue correlation ρ in our baseline scenario for $\rho \in [0.53, 0.93]$ evenly-spaced around the baseline value $\hat{\rho} = 0.73$ with a step size of 0.05. Paralleling Proposition 4, the optimal total expected profit decreases in ρ.

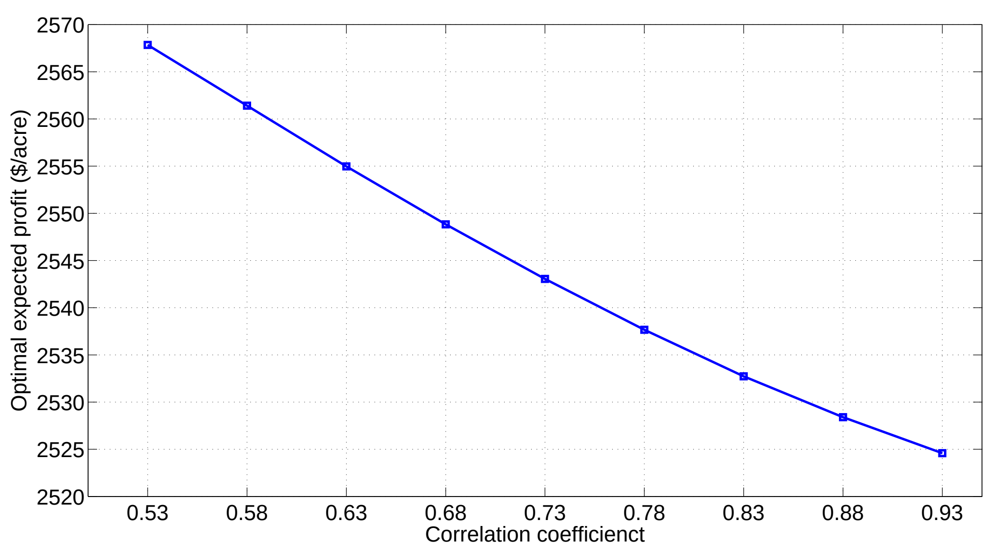

Figure 2: Impact of revenue correlation (ρ) on the farmer's optimal total expected profit over the planning horizon for $\rho \in [0.53, 0.93]$ evenly-spaced around the baseline value $\hat{\rho} = 0.73$ with a step size of 0.05.

Figure 3 illustrates the impact of soybean volatility σ^s in our baseline scenario for $\sigma^s \in [-50\%, 50\%]$ of the baseline value $\hat{\sigma}^s = 79.69$ with 5% increments. We observe that, paralleling Proposition 5, the optimal total expected profit decreases (increases) in soybean volatility when this volatility is low (high).

Figure 4 illustrates the impact of corn volatility σ^c in our baseline scenario for $\sigma^c \in [-50\%, 50\%]$ of the baseline value $\hat{\sigma}^c = 108.22$ with 5% increments. We observe that the optimal total expected profit always increases in corn volatility. When lower σ^c values—beyond -50% of the baseline value—are considered (not reported here), paralleling Proposition 5, we observe that the total expected profit first decreases then increases in σ^c. However, for realistic values of σ^c only increasing behavior is relevant as depicted in Figure 4.

In summary, we find that a typical farmer growing corn and soybean in Iowa benefits from a lower correlation between corn and soybean revenues. While the farmer always benefits from a higher corn volatility, a higher soybean revenue volatility is beneficial only when this volatility is sufficiently high; otherwise a lower soybean volatility is beneficial. These results highlight that there are significant differences between corn and soybean based on how revenue uncertainty of each crop shapes the farmer's profitability.

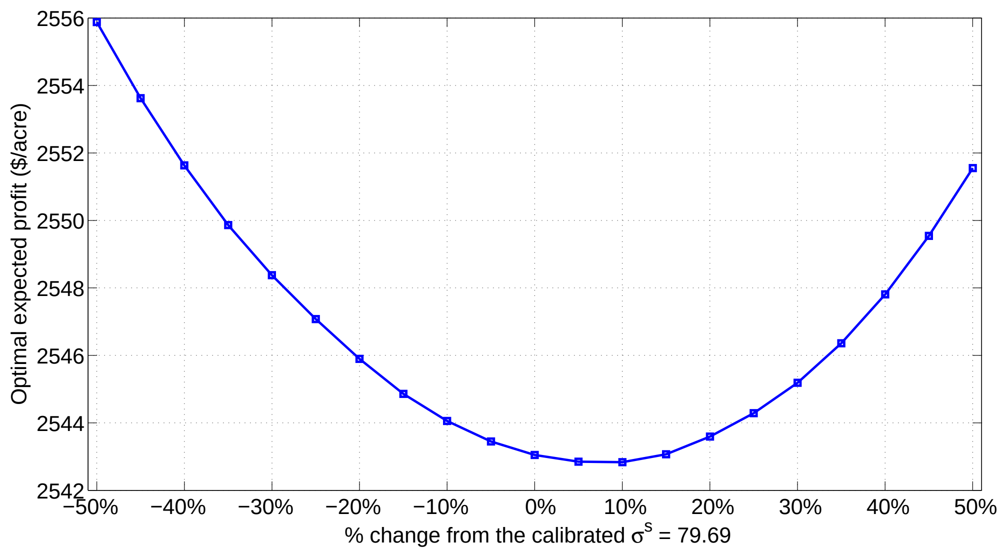

Figure 3: Impact of soybean revenue volatility (σ^s) on the farmer's optimal total expected profit over the planning horizon where $\sigma^s \in [-50\%, 50\%]$ of the baseline value $\hat{\sigma}^s = 79.69$ with 5% increments.

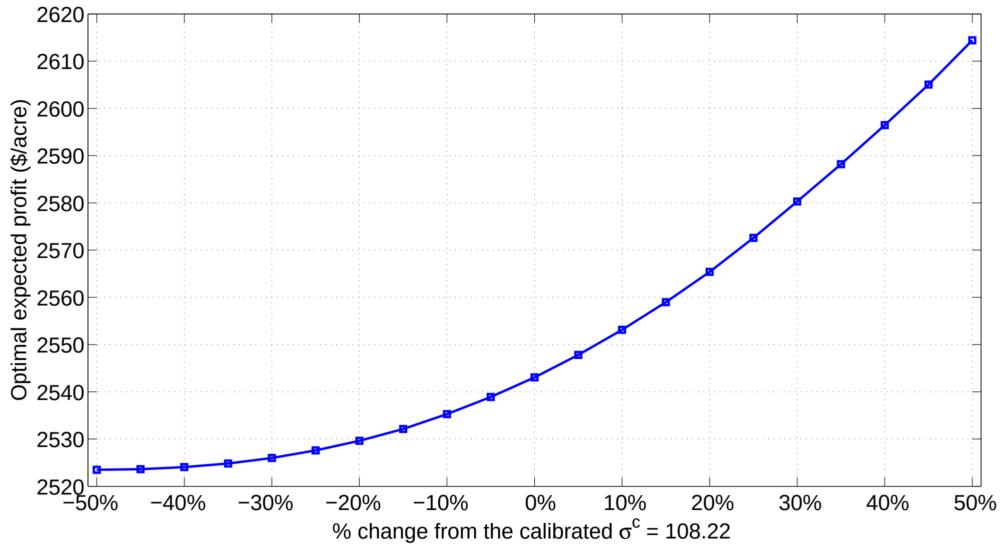

Figure 4: Impact of corn revenue volatility (σ^c) on the farmer's optimal total expected profit over the planning horizon where $\sigma^c \in [-50\%, 50\%]$ of the baseline value $\hat{\sigma}^c = 108.22$ with 5% increments.

5.2.3 Value of Sustainable Crop Planning

We now examine the value of making crop planning based on multiple crops with rotation benefits, as employed in sustainable agriculture, in comparison with continuously growing only one of the crops, as employed in industrial agriculture. To this end, we consider the benchmark case in which the farmer grows the same crop over the entire planning horizon. We define the profit loss due to continuously growing the same crop as

$$\Delta \doteq \left[\frac{V_1(\alpha_0, \boldsymbol{r_0}) - V_1^B(\alpha_0, \boldsymbol{r_0})}{V_1(\alpha_0, \boldsymbol{r_0})} \right],$$

where $V_1(\alpha_0, \boldsymbol{r_0})$ is the optimal total expected profit over the planning horizon and $V_1^B(\alpha_0, \boldsymbol{r_0})$ denotes the expected profit under the benchmark case. In the benchmark case the farmer has two options, growing corn or growing soybean over the entire planning horizon. Here, $V_1^B(\alpha_0, \boldsymbol{r_0})$ denotes the maximum expected profit from these two options.

We numerically compute the percentage profit loss $\Delta * 100$. To this end, we extend our numerical instances around the baseline scenario to consider sensitivity of our results based on several key parameters. In particular, we consider revenue correlation $\rho = \{0.53, 0.63, 0.73, 0.83, 0.93\}$, evenly-spaced around the baseline value 0.73; we consider corn (soybean) volatility σ^c (σ^s) that are $\{-50\%, -25\%, 0\%, 25\%, 50\%\}$ of their baseline values.

We also consider yield-enhancing rotation benefit b^j for crop $j \in \{c, s\}$ and cost-reducing rotation benefit γ^c for corn that are $\{-50\%, -25\%, 0\%, 25\%, 50\%\}$ of their baseline values (we continue to assume $\gamma^s = 0$). We set initial corn allocation $\alpha_0 \in \{0.38, 0.48, 0.58, 0.68, 0.78\}$, evenly-spaced around the baseline value 0.58. We also use different planning horizons, specifically $T \in \{5, 10, 15, 20\}$. In summary, we consider 312,500 numerical instances.

We find that the average profit loss in the numerical instances considered is 18.67% with a minimum and a maximum loss of 9.68% and 27.12%, respectively. This result indicates that making crop planning based on principles of sustainable agriculture has substantial economic value.[4]

5.2.4 Analysis of Heuristic Allocation Policies

In this section, we study the performance of a variety of heuristic allocation policies in comparison with the optimal policy. To this end, we numerically compute percentage profit loss $\Delta^H * 100$ due to employing heuristic policy (H). Here, Δ^H is defined as

$$\Delta^H \doteq \left[\frac{V_1(\alpha_0, \boldsymbol{r_0}) - V_1^H(\alpha_0, \boldsymbol{r_0})}{V_1(\alpha_0, \boldsymbol{r_0})}\right],$$

where $V_1(\alpha_0, \boldsymbol{r_0})$ is the optimal total expected profit over the planning horizon and $V_1^H(\alpha_0, \boldsymbol{r_0})$ denotes the expected profit under the heuristic allocation policy. We use the same 312,500 numerical instances as in §5.2.3.

We restrict our attention to heuristic policies in which the periodic allocation decision, as denoted by α_t^H for $t \in \{1, T\}$, can be characterized in closed form, and thus, that are easily implementable in practice. In particular, we consider the following heuristic policies:

1) **Always Rotate.** Under this policy the farmer grows each crop only on rotated farmland in each period , i.e., $\alpha_t^H = 1 - \alpha_{t-1} \in \{\alpha_0, 1 - \alpha_0\}$ for $t \in [1, T]$. We consider this policy based on our conversations with a farmer growing corn and soybean in the U.S.

2) **Always Rotate (Monoculture).** Under this policy the farmer grows only one of the crops in each period and rotates to the other crop in the subsequent period. This is a commonly suggested heuristic policy in the literature (see, for example, Livingston et al. 2015). We consider two heuristics based on the crop choice in the first period: The

[4]There may exist additional costs associated with growing multiple crops instead of a single crop which are not considered in our model. For example, each crop may require a different set of management skills or machinery for cultivation and harvesting. Nevertheless, our analysis provides the benefit of growing multiple crops instead of a single crop which can later be compared with these additional costs.

	Always Rotate	Always Rotate (Monoculture)	Myopic	One-period Lookahead
Average	1.13%	1.85%	0.80%	0.03%
Min	0.23%	0.60%	0.17%	0.00%
Max	3.83%	4.09%	2.20%	0.13%

Table 2: Performance of heuristic allocation policies. For each heuristic (H), "Average" denotes the average percentage profit loss ($\Delta^H * 100$), whereas "Min" and "Max" denote the minimum and the maximum percentage profit loss observed in all numerical instances.

farmer first grows corn i.e., $\alpha_1^H = 1$, or the farmer first grows soybean, i.e., $\alpha_1^H = 0$. In both cases the allocation in the rest of the planning horizon ($t \in [2, T]$) is given by $\alpha_t^H = 1 - \alpha_t \in \{0, 1\}$. In each numerical instance, we only report the better performing heuristic—that is, $V_1^H(\alpha_0, \boldsymbol{r_0})$ denotes the highest expected profit of the two heuristics.

3) **Myopic.** Under this policy the farmer chooses the allocation in each period ignoring the cash flows from future periods. The optimal allocation in period $t \in [1, T]$ under the myopic policy can be obtained from (3) of Proposition 1 by substituting $t = 2$ with an arbitrary t, and using the identity $\mathbb{E}_t[\tilde{r}_t^j] = e^{-\kappa^j} r_{t-1}^j + \left(1 - e^{-\kappa^j}\right) \xi^j$ for $j \in \{c, s\}$.

4) **One-period Lookahead.** Under this policy the farmer chooses the allocation in period t based on a two-period horizon—that is, by considering the future cash flows only from the subsequent period $t+1$. The optimal allocation in period $t \in [1, T]$ under this policy can be obtained from (5) of Proposition 2 by substituting $t = 1$ ($t-1 = 0$, $t+1 = 2$) with an arbitrary t ($t-1$, $t+1$), and using the identities given in Proposition 3.

Two remarks are in order. First, although myopic and one-period lookahead policies are classical heuristic policies used in multi-period models, the closed-form characterization of the optimal periodic allocation decision under each heuristic is only made possible by our theoretical analysis in §4. Second, among the four heuristic policies considered, only under the one-period lookahead policy the optimal periodic allocation decision is affected by the revenue volatility of each crop and the revenue correlation between the two crops.

Table 2 summarizes the average, minimum, and maximum percentage profit loss $\Delta^H * 100$ observed under each heuristic policy in all numerical instances. We make the following important observations:

1) We observe in all numerical instances that the profit loss is the smallest with the one-period lookahead policy and the maximum percentage loss with this policy is only 0.13% as reported in Table 2. In other words, *the one-period lookahead policy not only outperforms the commonly suggested heuristic policies in the literature but also provides a near-optimal performance.* The one-period lookahead policy outperforms the other heuristic policies because in making the allocation decision in each period, unlike other policies, this policy uses the information about revenue uncertainty—i.e., revenue volatility of each crop and revenue correlation between crops—which is a critical feature of growing corn and soybean as discussed in §1. The performance of the one-period lookahead policy is very close to the performance of the optimal policy because (i) by assumption (one-period carrying through of crop rotation benefits) the allocation decision in each period only impacts the allocation decision in the subsequent period, and (ii) this impact is captured by the one-period lookahead policy owing to the two-period horizon considered.

2) In all numerical instances the profit loss with the Always Rotate (Monoculture) policy is larger than the Always Rotate policy. In the literature, papers studying the farmland allocation decision in a multi-period setting (for example, Livingston et al. 2015) suggest farmers to use rotation-based allocation policy. Because these papers assume growing of a single crop every season their proposed policy corresponds to Always Rotate (Monoculture) in our framework. Our results underline the importance of considering the possibility of growing more than one crop in the same season in employing a rotation-based allocation policy (as in the case of Always Rotate policy): *When the farmer chooses to follow a rotation-based allocation policy, there is value in considering the possibility of growing more than one crop in the same season.*

6 Conclusion

This paper examines crop planning decision in sustainable agriculture—that is, how to allocate farmland among multiple crops in each growing season when the crops have rotation benefits across growing seasons. Because farmers in practice face significant uncertainty for their crop revenues we study the crop planning decision under revenue uncertainty. This is the first paper that characterizes the optimal dynamic farmland allocation policy under uncertainty in the presence of crop rotation benefits. As summarized in the Introduction, we provide insights on how revenue uncertainty of each crop shapes the farmer's optimal profit, and insights on the optimality gap when using heuristic policies suggested in the literature. Based on our optimal policy characterization we propose a simple sustainable heuristic allocation policy and show that the proposed policy not only outperforms the other suggested heuristic policies but also provides a near-optimal performance.

In our computational study throughout §5.2, we calibrated our model to represent a typical farmer growing corn and soybean in Iowa. We expect our insights to continue to hold for a farmer growing corn and soybean in another location (e.g., Illinois in the U.S., Brazil) or growing other crops with rotation benefits (e.g., cotton, wheat and rice). In particular, we expect the farmer to benefit from a lower (higher) revenue volatility when this volatility is low (high); but the actual range in which the volatility in practice falls depends on the specific crop considered. We also expect our proposed heuristic policy to outperform the other suggested heuristic policies in the literature. This is because, unlike those other policies, our suggested policy uses the information about revenue uncertainty—i.e., revenue volatility of each crop and revenue correlation between crops—which is a critical feature of crops with rotation benefits listed above. The performance of the proposed heuristic policy in comparison to the optimal policy again depends on the specific crops considered. Future research may re-calibrate our model to capture the characteristics of a typical farmer growing corn and soybean in another location or a farmer growing other crops with rotation benefits by using the methodology presented in this paper.

Our model can be used to study other interesting research questions. For instance, in line with the objectives of sustainable agriculture it is important to understand the environmental impact of crop planning decision. Because rotating crops improves the soil structure, reduces the need for synthetic chemicals, and increases the biodiversity in the farmland, one way to asses that environmental impact is to measure the prevalence of crop rotations in the farmer's optimal allocation policy. What is the average percentage of farmland allocated to rotated crops in a growing season over the planning horizon? How is it affected by key parameters (e.g., revenue uncertainty)? In practice farmers are supported by a variety of government programs including crop-specific subsidies (e.g., increasing availability of seeds), generic subsidies (e.g., direct payments to farmers, subsidies for irrigation water), and regulations that set minimum prices for crops (Kazaz et al. 2014). A related research question is to examine the effectiveness of those government programs in increasing the prevalence of crop rotations in the optimal policy.

Relaxing the assumptions made about the crop features gives rise to a number of interesting areas for future research. First, we assume that farmland is always fully allocated to the available crops in each growing season and we only consider two crop choices. In practice farmers may let the farmland lie fallow in a growing season to rejuvenate the soil (Taylor and Burt 1984) or they may consider more than two crop choices. Second, we assume that crop rotation benefits are exhausted after one period. Although this is a reasonable assumption for some crop rotations, including corn-soybean rotation as empirically documented in Hennessy (2006), it can be a limitation for others. Third, there may be constraints on the farmland allocated to each crop in a growing season owing to, for example, limited availability of crop-specific resources (e.g., seeds) or industry regulations that encourage growing of a particular crop in that season (Whan et al. 1978).

Our model (implicitly) assumes that the farmer's crop planning decision has no effect on crop revenues, an assumption that is commonly made in the literatures of operations management and agricultural economics. This is a reasonable assumption for commodity crops—including corn and soybean as considered in this paper—where production volume of an individual farmer is insignificant in comparison to the aggregate production volume that are traded in the exchange (spot) markets. For other crops the farmer's crop planning decision may affect the crop revenue by altering its availability in the market. Examining the crop planning decision in this setting requires an equilibrium model that captures the interplay between crop availability and crop revenue, following the work of Mendelson and Tunca (2007) who provide an equilibrium model in the context of commodity procurement.

Finally, we restrict our attention to crop planning decision. Crop production however involves subsequent operational decisions during cultivation (e.g., fertilizer and pesticides application, irrigation planning) and harvesting (e.g., harvest timing). Those operational decisions have an impact on crop revenues which we assume uncertain but exogenous in our model. Combining the crop planning decision with those other operational decisions in crop production is beyond the scope of this paper and should prove to be an interesting avenue for future research.

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Appendix

We use the following notation and results throughout the appendix. Let $\phi(.)$ and $\Phi(.)$ denote the p.d.f and c.d.f. of the standard normal random variable, respectively. $\phi^{'}(z) = -z\phi(z)$ and $\phi(z) = \phi(-z)$. $\ln x$ denotes natural logarithm of x. We use the following result from Cain (1994):

Lemma 1 *Let $\mathbf{X} = (X_1, X_2)$ follow a bivariate normal distribution with mean vector $\boldsymbol{\mu} = (\mu_1, \mu_2)$, and covariance matrix $\boldsymbol{\Sigma}$ where $\boldsymbol{\Sigma}_{jj} = \sigma_j^2$ for $j = 1, 2$ and $\boldsymbol{\Sigma}_{12} = \rho\sigma_1\sigma_2$ and ρ denotes the correlation coefficient.*

$$\mathbb{E}[\max(X_1, X_2)] = \mu_1 \Phi\left(\frac{\mu_1 - \mu_2}{\theta}\right) + \mu_2 \Phi\left(\frac{\mu_2 - \mu_1}{\theta}\right) + \theta\phi\left(\frac{\mu_2 - \mu_1}{\theta}\right),$$

where $\theta \doteq \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$.

We also use the following result proof of which is omitted for brevity.

Lemma 2 *For $-1 < y \leq 0$, $\frac{y}{2}\left(\frac{2+y}{1+y}\right) \leq \ln(1+y)$.*

Proof of Proposition 1: See the proof of the T-period problem in Proposition 6. ■

Proof of Proposition 2: See the proof of the T-period problem in Proposition 6. ■

Proof of Proposition 3: The expressions for $K_2^j(\boldsymbol{r_1})$ are obtained from (4) by using $\mathbb{E}_2[\tilde{r}_2^j] = e^{-\kappa^j} r_1^j + \left(1 - e^{-\kappa^j}\right) \xi^j$ for $j \in \{c, s\}$. Given $\boldsymbol{r_0} = (r_0^c, r_0^s)$, $(\tilde{r}_1^c, \tilde{r}_1^s)$ follow a bi-variate normal distribution with mean vector (μ_1^c, μ_1^s), and covariance matrix $\boldsymbol{\Sigma}$ with $\boldsymbol{\Sigma}_{11} = \text{VAR}_1^c$, $\boldsymbol{\Sigma}_{22} = \text{VAR}_1^s$ and $\boldsymbol{\Sigma}_{12} = \text{COV}_1$ where $\mu_1^j \doteq \mathbb{E}[\tilde{r}_1^j|\boldsymbol{r_0}]$, $\text{VAR}_1^j \doteq \text{VAR}[\tilde{r}_1^j|\boldsymbol{r_0}]$, and $\text{COV}_1 \doteq \text{COV}[\tilde{r}_1^c, \tilde{r}_1^s|\boldsymbol{r_0}]$ are given by (8) with $t = 1$ and $\hat{t} = 0$. Therefore, $(\underline{u}\tilde{r}_1^j + \underline{v}, \overline{u}\tilde{r}_1^{-j} + \overline{v})$ follow a bi-variate normal distribution with mean vector $(\underline{u}\mu_1^j + \underline{v}, \overline{\mu}_1^{-j} + \overline{v})$ and covariance matrix $\hat{\boldsymbol{\Sigma}}$ with $\hat{\boldsymbol{\Sigma}}_{11} = \underline{u}^2\text{VAR}_1^j$, $\hat{\boldsymbol{\Sigma}}_{22} = \overline{u}^2\text{VAR}_1^{-j}$, and $\hat{\boldsymbol{\Sigma}}_{12} = \underline{u}\overline{u}\text{COV}_1$. The identity in (9) follows from Lemma 1. ■

Proof of Proposition 4: From Proposition 2, $V_1(\alpha_0, \boldsymbol{r_0}) = \alpha_0 K_1^c(\boldsymbol{r_0}) + (1 - \alpha_0) K_1^s(\boldsymbol{r_0})$ where $K_1^j(\boldsymbol{r_0})$ for $j \in \{c, s\}$ is as given in (6). Under the bi-variate normal distribution specified in (8), $\mathbb{E}_1[\tilde{r}_1^j] = e^{-\kappa^j} r_0^j + \left(1 - e^{-\kappa^j}\right) \xi^j$ and thus $K_1^j(\boldsymbol{r_0})$ is impacted by ρ through its effect on $\mathbb{E}_1[K_2^c(\tilde{\boldsymbol{r}}_1)]$ and $\mathbb{E}_1[K_2^s(\tilde{\boldsymbol{r}}_1)]$. From Proposition 3, $\mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)] = \mathbb{E}_1[\max\{\underline{u}r_1^j + \underline{v}, \overline{u}r_1^{-j} + \overline{v}\}]$ for $j \in \{c, s\}$ where $\underline{u} = e^{-\kappa^j}$, $\underline{v} = \left(1 - e^{-\kappa^j}\right)\xi^j - f^j$, $\overline{u} = (1 + b^{-j})e^{-\kappa^{-j}}$, $\overline{v} = (1 + b^{-j})\left(1 - e^{-\kappa^{-j}}\right)\xi^{-j} - (1 - \gamma^{-j})f^{-j}$. Using the identity in (9), we obtain

$$\frac{\partial \mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]}{\partial \rho} = \frac{\partial \mathbb{E}_1[\max\{\underline{u}\tilde{r}_1^j + \underline{v}, \overline{u}\tilde{r}_1^{-j} + \overline{v}\}]}{\partial \rho} = \phi\left(\frac{\overline{u}\mu^{-j} + \overline{v} - \underline{u}\mu^j - \underline{v}}{\lambda}\right)\frac{\partial \lambda}{\partial \rho},$$

where $\frac{\partial\lambda}{\partial\rho} = -\frac{\underline{u}\overline{u}}{\lambda}\frac{\partial \text{COV}_1}{\partial\rho} < 0$ because $\lambda = \sqrt{\underline{u}^2\text{VAR}_1^c + \overline{u}^2\text{VAR}_1^s - 2\underline{u}\overline{u}\text{COV}_1} > 0$ and $\frac{\partial \text{COV}_1}{\partial\rho} = \frac{1-e^{-(\kappa^c+\kappa^s)}}{\kappa^c+\kappa^s}\sigma^c\sigma^s > 0$. Therefore, $\frac{\partial\mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]}{\partial\rho} < 0$, and thus $\frac{\partial K_1^j(\tilde{\boldsymbol{r}}_1)}{\partial\rho} < 0$. ■

Proof of Proposition 5: It is easy to establish that

$$\begin{aligned}
K_1^c(\boldsymbol{r_0}) &\doteq \max\left\{-f^c + \mathbb{E}_1[\tilde{r}_1^c] + \mathbb{E}_1[K_2^c(\tilde{\boldsymbol{r}}_1)], -(1-\gamma^s)f^s + \mathbb{E}_1[(1+b^s)\tilde{r}_1^s] + \mathbb{E}_1[K_2^s(\tilde{\boldsymbol{r}}_1)]\right\}, \\
K_1^s(\boldsymbol{r_0}) &\doteq \max\left\{-(1-\gamma^c)f^c + \mathbb{E}_1[(1+b^c)\tilde{r}_1^c] + \mathbb{E}_1[K_2^c(\tilde{\boldsymbol{r}}_1)], -f^s + \mathbb{E}_1[\tilde{r}_1^s] + \mathbb{E}_1[K_2^s(\tilde{\boldsymbol{r}}_1)]\right\}
\end{aligned}$$

are impacted by σ^j for $j \in \{c, s\}$ through its effect on $\mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]$ and $\mathbb{E}_1[K_2^{-j}(\tilde{\boldsymbol{r}}_1)]$. To prove the impact of σ^j on $V_1(\alpha_0, \boldsymbol{r_0}) = \alpha_0 K_1^c(\boldsymbol{r_0}) + (1-\alpha_0)K_1^s(\boldsymbol{r_0})$ we will establish the following arguments:

(i) $\mathbb{E}_1[K_2^c(\tilde{\boldsymbol{r}}_1)]$ and $\mathbb{E}_1[K_2^s(\tilde{\boldsymbol{r}}_1)]$ first decrease then increase in σ^j,

(ii) $\mathbb{E}_1[K_2^c(\tilde{\boldsymbol{r}}_1)]$ and $\mathbb{E}_1[K_2^s(\tilde{\boldsymbol{r}}_1)]$ are convex in σ^j.

Because convexity is preserved under maximization (ii) implies that $K_1^c(\boldsymbol{r_0})$ and $K_1^s(\boldsymbol{r_0})$ are also convex in σ^j, and thus $V_1(\alpha_0, \boldsymbol{r_0}) = \alpha_0 K_1^c(\boldsymbol{r_0}) + (1-\alpha_0)K_1^s(\boldsymbol{r_0})$ is also convex in σ^j. Moreover, it follows from (i) that $\mathbb{E}_1[K_2^c(\tilde{\boldsymbol{r}}_1)]$ and $\mathbb{E}_1[K_2^s(\tilde{\boldsymbol{r}}_1)]$, and thus $V_1(\alpha_0, \boldsymbol{r_0})$ decrease in σ^j for sufficiently small σ^j and increase in σ^j for sufficiently large σ^j. because $V_1(\alpha_0, \boldsymbol{r_0})$ is convex in σ^j, there exists a unique $\overline{\sigma}^j$ such that $\frac{\partial V_1(\alpha_0,\boldsymbol{r_0})}{\partial\sigma^j} \le 0$ if $\sigma^j < \overline{\sigma}^j$; and $\frac{\partial V_1(\alpha_0,\boldsymbol{r_0})}{\partial\sigma^j} \ge 0$ if $\sigma^j > \overline{\sigma}^j$.

We now provide the proof for (i). Following similar steps with Proposition 4, we obtain

$$\frac{\partial\mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]}{\partial\sigma^j} = \frac{\partial\mathbb{E}_1[\max\{\underline{u}\tilde{r}_1^j + \underline{v}, \overline{u}\tilde{r}_1^{-j} + \overline{v}\}]}{\partial\sigma^j} = \phi\left(\frac{\overline{u}\mu^{-j} + \overline{v} - \underline{u}\mu^j - \underline{v}}{\lambda}\right)\frac{\partial\lambda}{\partial\sigma^j}, \tag{14}$$

where

$$\frac{\partial\lambda}{\partial\sigma^j} = \frac{1}{\lambda}\left[\underline{u}^2\frac{1-e^{-2\kappa^j}}{2\kappa^j}\sigma^j - \underline{u}\overline{u}\frac{1-e^{-(\kappa^j+\kappa^{-j})}}{\kappa^j+\kappa^{-j}}\rho\sigma^{-j}\right]. \tag{15}$$

The term inside the bracket can be written as $A\sigma^j - B$, where $A \doteq \underline{u}^2\frac{1-e^{-2\kappa^j}}{2\kappa^j} > 0$ and $B \doteq \underline{u}\overline{u}\frac{1-e^{-(\kappa^j+\kappa^{-j})}}{\kappa^j+\kappa^{-j}}\rho\sigma^{-j} > 0$. Therefore there exists a unique $\hat{\sigma}^j \doteq \frac{B}{A}$ threshold such that $\frac{\partial\mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]}{\partial\sigma^j} < 0$ for $\sigma^j < \hat{\sigma}^j$ and $\frac{\partial\mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]}{\partial\sigma^j} > 0$ for $\sigma^j > \hat{\sigma}^j$. Following similar steps, it can be established that there exists a unique $\hat{\hat{\sigma}}^j$ threshold such that $\frac{\partial\mathbb{E}_1[K_2^{-j}(\tilde{\boldsymbol{r}}_1)]}{\partial\sigma^j} < 0$ for $\sigma^j < \hat{\hat{\sigma}}^j$ and $\frac{\partial\mathbb{E}_1[K_2^{-j}(\tilde{\boldsymbol{r}}_1)]}{\partial\sigma^j} > 0$ for $\sigma^j > \hat{\hat{\sigma}}^j$.

To conclude we now provide the proof for (ii). We will only show the convexity of $\mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]$ in σ^j, i.e., $\frac{\partial^2\mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]}{\partial(\sigma^j)^2} \ge 0$; the convexity of $\mathbb{E}_1[K_2^{-j}(\tilde{\boldsymbol{r}}_1)]$ in σ^j can be established in a similar fashion. For expositional purposes, besides $A = \underline{u}^2\frac{1-e^{-2\kappa^j}}{2\kappa^j}$ and

$B = \underline{u}\overline{u}\frac{1-e^{-(\kappa^j+\kappa^{-j})}}{\kappa^j+\kappa^{-j}}\rho\sigma^{-j}$, we define $a \doteq \overline{u}\mu^{-j} + \overline{v} - \underline{u}\mu^j - \underline{v}$ and $C \doteq \overline{u}^2\frac{1-e^{-2\kappa^{-j}}}{2\kappa^{-j}}(\sigma^{-j})^2$. Using this notation, it follows from (14) and (15) that

$$\frac{\partial \mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]}{\partial \sigma^j} = \frac{\partial \mathbb{E}_1[\max\{\underline{u}\tilde{r}_1^j + \underline{v}, \overline{u}\tilde{r}_1^{-j} + \overline{v}\}]}{\partial \sigma^j} = \phi\left(\frac{a}{\lambda}\right)\left(\frac{A\sigma^j - B}{\lambda}\right),$$

where $\lambda = \sqrt{A(\sigma^j)^2 + C - 2B\sigma^{-j}}$. We obtain

$$\frac{\partial^2 \mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]}{\partial (\sigma^j)^2} = \phi\left(\frac{a}{\lambda}\right)\frac{1}{\lambda}\left[\frac{A\sigma^j - B}{\lambda}\right]^2\left[\frac{a}{\lambda}\right]^2 + \phi\left(\frac{a}{\lambda}\right)\frac{1}{\lambda}\left[A - \left(\frac{A\sigma^j - B}{\lambda}\right)^2\right].$$

To prove $\frac{\partial^2 \mathbb{E}_1[K_2^j(\tilde{\boldsymbol{r}}_1)]}{\partial (\sigma^j)^2} \geq 0$, it is sufficient to show that $A - \left(\frac{A\sigma^j - B}{\lambda}\right)^2 \geq 0$. Using the definitions of A, B and λ, it is easy to establish that this condition is equivalent to

$$\left(\frac{1-e^{-2\kappa^j}}{2\kappa^j}\right)\left(\frac{1-e^{-2\kappa^{-j}}}{2\kappa^{-j}}\right) \geq \left(\frac{1-e^{-(\kappa^j+\kappa^{-j})}}{\kappa^j+\kappa^{-j}}\right)^2 \rho^2.$$

Because $\rho \leq 1$, it is sufficient to show

$$\left(\frac{1-e^{-2\kappa^j}}{2\kappa^j}\right)\left(\frac{1-e^{-2\kappa^{-j}}}{2\kappa^{-j}}\right) \geq \left(\frac{1-e^{-(\kappa^j+\kappa^{-j})}}{\kappa^j+\kappa^{-j}}\right)^2. \tag{16}$$

The condition in (16) is satisfied with equality for $\kappa^j = \kappa^{-j}$; therefore we focus on the case $\kappa^j \neq \kappa^{-j}$ hereafter. We define $u \doteq e^{-\kappa^j}$ and $v \doteq e^{-\kappa^{-j}}$ where $0 < u, v < 1$, $\kappa^j = -\ln u$, and $\kappa^{-j} = -\ln v$. After some algebra, (16) can be equivalently written as

$$\left(\frac{1-uv}{\ln uv}\right)^2 \geq \left(\frac{u-v}{\ln u - \ln v}\right)^2. \tag{17}$$

In (17), the right-hand side $l \doteq \frac{u-v}{\ln u - \ln v}$ is the logarithmic mean of u and v. It follows from the ordering among the geometric, logarithmic and arithmetic means (see, for example, Burk 1987) that $\sqrt{uv} \leq l \leq \frac{u+v}{2}$. Using $\sqrt{uv} \leq l$, to prove (17), it suffices to show

$$\left(\frac{1-l^2}{\ln l^2}\right)^2 \geq l^2. \tag{18}$$

Because $u, v < 1$, we have $\frac{u+v}{2} < 1$ and so $l \leq \frac{u+v}{2} < 1$ from the ordering of logarithmic and arithmetic means. Therefore, $1 - l^2 > 0$ and $\ln l < 0$. It follows that the condition in (18) is equivalent to $\ln l \geq \frac{l^2-1}{2l}$ which is satisfied because, as follows from Lemma 2

$$\ln l = \ln(1 + (l-1)) \geq \frac{l-1}{2}\left(\frac{2 + (l-1)}{1 + (l-1)}\right) = \frac{l^2-1}{2l},$$

where $-1 < l - 1 < 0$ as $0 < l < 1$. This concludes the proof. ■

Proof of Proposition 6: Let $\pi_t(\alpha_t) \doteq L(\alpha_t \mid \alpha_{t-1}, \boldsymbol{r_{t-1}}) + \mathbb{E}_t[V_{t+1}(\alpha_t, \tilde{\boldsymbol{r}}_{\boldsymbol{t}})]$ denote the objective function of the optimization problem in period $t \in [1, T]$ where the immediate payoff $L(\alpha_t \mid \alpha_{t-1}, \boldsymbol{r_{t-1}})$ is as given in (1) and the optimal value function from period $t+1$ onwards $V_{t+1}(\alpha_t, \tilde{\boldsymbol{r}}_{\boldsymbol{t}})$ is as given in (2). We provide a proof by induction. Consider first $t = T$. Because $V_{T+1}(\cdot) = 0$, $\pi_T(\alpha_T) = L(\alpha_T \mid \alpha_{T-1}, \boldsymbol{r_{T-1}})$. We obtain

$$\frac{\partial \pi_T(\alpha_T)}{\partial \alpha_T} = \begin{cases} h_T^1 \doteq (1+b^c)\mathbb{E}_T[\tilde{r}_T^c] - (1-\gamma^c)f^c - \mathbb{E}_T[\tilde{r}_T^s] + f^s & \text{if } 0 \leq \alpha_T \leq 1-\alpha_{T-1}, \\ h_T^2 \doteq \mathbb{E}_T[\tilde{r}_T^c] - f^c - (1+b^s)\mathbb{E}_T[\tilde{r}_T^s] + (1-\gamma^s)f^s & \text{if } 1-\alpha_{T-1} < \alpha_T \leq 1, \end{cases}$$

where $h_T^1 - h_T^2 = \Sigma_j \left(b^j \mathbb{E}_T[\tilde{r}_T^j] + \gamma^j f^j \right) > 0$. Therefore, π_T is piecewise linear and concave in α_T with a kink at $\alpha_T = 1 - \alpha_{T-1}$. We have three cases to consider:

(i) $\frac{\partial \pi_T}{\partial \alpha_T}\Big|_{0^+} \leq 0$, i.e., $h_T^1 \leq 0$. In this case, $\alpha_T^* = 0$ and $\pi_T(0) = -f^s + \mathbb{E}_T[\tilde{r}_T^s] + \alpha_{T-1}(b^s\mathbb{E}_T[\tilde{r}_T^s] + \gamma^s f^s)$. Because $h_T^1 \leq 0$, $K_T^j(\boldsymbol{r_{T-1}})$ for $j \in \{c, s\}$ as defined in (11) is such that $K_T^c(\boldsymbol{r_{T-1}}) = (1+b^s)\mathbb{E}_T[\tilde{r}_T^s] - (1-\gamma^s)f^s$ and $K_T^s(\boldsymbol{r_{T-1}}) = \mathbb{E}_T[\tilde{r}_T^s] - f^s$. It is easy to establish that $\alpha_{T-1}K_T^c(\boldsymbol{r_{T-1}}) + (1-\alpha_{T-1})K_T^s(\boldsymbol{r_{T-1}})$ equals $\pi_T(0)$ and thus the characterization of $V_T(\alpha_{T-1}, \boldsymbol{r_{T-1}})$ as given in (12) holds.

(ii) $\frac{\partial \pi_T}{\partial \alpha_T}\Big|_{1^-} \geq 0$, i.e., $h_T^2 \geq 0$. In this case, $\alpha_T^* = 1$ and $\pi_T(1) = -f^c + \mathbb{E}_T[\tilde{r}_T^c] + (1-\alpha_{T-1})(b^c\mathbb{E}_T[\tilde{r}_T^c] + \gamma^c f^c)$. Because $h_T^2 \geq 0$, $K_T^j(\boldsymbol{r_{T-1}})$ for $j \in \{c, s\}$ as defined in (11) is such that $K_T^c(\boldsymbol{r_{T-1}}) = \mathbb{E}_T[\tilde{r}_T^c] - f^c$ and $K_T^s(\boldsymbol{r_{T-1}}) = (1+b^c)\mathbb{E}_T[\tilde{r}_T^c] - (1-\gamma^c)f^c$. It is easy to establish that $\alpha_{T-1}K_T^c(\boldsymbol{r_{T-1}}) + (1-\alpha_{T-1})K_T^s(\boldsymbol{r_{T-1}})$ equals $\pi_T(1)$ and thus the characterization of $V_T(\alpha_{T-1}, \boldsymbol{r_{T-1}})$ as given in (12) holds.

(iii) $\frac{\partial \pi_T}{\partial \alpha_T}\Big|_{0^+} > 0$ and $\frac{\partial \pi_T}{\partial \alpha_T}\Big|_{1^-} < 0$, i.e., $h_T^1 > 0$ and $h_T^2 < 0$. In this case, $\alpha_T^* = 1 - \alpha_{T-1}$ and $\pi_T(1-\alpha_{T-1}) = \alpha_{T-1}\left(-(1-\gamma^s)f^s + (1+b^s)\mathbb{E}_T[\tilde{r}_T^s]\right) + (1-\alpha_{T-1})\left(-(1-\gamma^c)f^c + (1+b^c)\mathbb{E}_T[\tilde{r}_T^c]\right)$. Because $h_T^1 > 0$ and $h_T^2 < 0$, $K_T^j(\boldsymbol{r_{T-1}})$ for $j \in \{c, s\}$ as defined in (11) is such that $K_T^c(\boldsymbol{r_{T-1}}) = -(1-\gamma^s)f^s + (1+b^s)\mathbb{E}_T[\tilde{r}_T^s]$ and $K_T^s(\boldsymbol{r_{T-1}}) = -(1-\gamma^c)f^c + (1+b^c)\mathbb{E}_T[\tilde{r}_T^c]$. It follows that $\alpha_{T-1}K_T^c(\boldsymbol{r_{T-1}}) + (1-\alpha_{T-1})K_T^s(\boldsymbol{r_{T-1}})$ equals $\pi_T(1-\alpha_{T-1})$ and thus the characterization of $V_T(\alpha_{T-1}, \boldsymbol{r_{T-1}})$ as given in (12) holds.

This concludes that the characterizations of α_t^* in (10) and $V_t(\alpha_{t-1}, \boldsymbol{r_{t-1}})$ in (12) hold for $t = T$. As the second step in induction, we assume that these characterizations also hold for period $t+1$ and prove that they also hold for period t. We have $\pi_t(\alpha_t) \doteq L(\alpha_t \mid \alpha_{t-1}, \boldsymbol{r_{t-1}}) + \mathbb{E}_t[V_{t+1}(\alpha_t, \tilde{\boldsymbol{r}}_{\boldsymbol{t}})]$ where $\mathbb{E}_t[V_{t+1}(\alpha_t, \tilde{\boldsymbol{r}}_{\boldsymbol{t}})] = \alpha_t K_{t+1}^c(\boldsymbol{r_t}) + (1-\alpha_t)K_{t+1}^s(\boldsymbol{r_t})$ by the induction assumption. We obtain

$$\frac{\partial \pi_t(\alpha_t)}{\partial \alpha_t} = \begin{cases} h_t^1 \doteq \mathbb{E}_t[(1+b^c)\tilde{r}_t^c + K_{t+1}^c(\boldsymbol{r_t})] - (1-\gamma^c)f^c - \mathbb{E}_t[\tilde{r}_t^s + K_{t+1}^s(\boldsymbol{r_t})] + f^s & \text{if } \; 0 \le \alpha_t \le 1-\alpha_{t-1}, \\ h_t^2 \doteq \mathbb{E}_t[\tilde{r}_t^c + K_{t+1}^c(\boldsymbol{r_t})] - f^c - \mathbb{E}_t[(1+b^s)\tilde{r}_t^s + K_{t+1}^s(\boldsymbol{r_t})] + (1-\gamma^s)f^s & \text{if } \; 1-\alpha_{t-1} < \alpha_t \le 1, \end{cases}$$

where $h_t^1 - h_t^2 = \Sigma_j \left(b^j \mathbb{E}_T[\tilde{r}_t^j] + \gamma^j f^j \right) > 0$. Therefore, π_t is piecewise linear and concave in α_t with a kink at $\alpha_t = 1 - \alpha_{t-1}$. We have three cases to consider:

(i) $\frac{\partial \pi_t}{\partial \alpha_t}\Big|_{0+} \le 0$, i.e., $h_t^1 \le 0$. In this case, $\alpha_t^* = 0$ and $\pi_t(0) = -f^s + \mathbb{E}_t[\tilde{r}_t^s + K_{t+1}^s(\boldsymbol{r_t})] + \alpha_{t-1}(b^s\mathbb{E}_t[\tilde{r}_t^s] + \gamma^s f^s)$. Because $h_t^1 \le 0$, $K_t^j(\boldsymbol{r_{t-1}})$ for $j \in \{c, s\}$ as defined in (11) is such that $K_t^c(\boldsymbol{r_{t-1}}) = \mathbb{E}_t[(1+b^s)\tilde{r}_t^s + K_{t+1}^s(\boldsymbol{r_t})] - (1-\gamma^s)f^s$ and $K_t^s(\boldsymbol{r_{t-1}}) = \mathbb{E}_t[\tilde{r}_t^s + K_{t+1}^s(\boldsymbol{r_t})] - f^s$. It is easy to establish that $\alpha_{t-1}K_t^c(\boldsymbol{r_{t-1}}) + (1-\alpha_{t-1})K_t^s(\boldsymbol{r_{t-1}})$ equals $\pi_t(0)$ and thus the characterization of $V_t(\alpha_{t-1}, \boldsymbol{r_{t-1}})$ as given in (12) holds.

(ii) $\frac{\partial \pi_t}{\partial \alpha_t}\Big|_{1-} \ge 0$, i.e., $h_t^2 \ge 0$. In this case, $\alpha_t^* = 1$ and $\pi_t(1) = -f^c + \mathbb{E}_t[\tilde{r}_t^c + K_{t+1}^c(\boldsymbol{r_t})] + (1-\alpha_{t-1})(b^c\mathbb{E}_t[\tilde{r}_t^c] + \gamma^c f^c)$. Because $h_t^2 \ge 0$, $K_t^j(\boldsymbol{r_{t-1}})$ for $j \in \{c, s\}$ as defined in (11) is such that $K_t^c(\boldsymbol{r_{t-1}}) = \mathbb{E}_t[\tilde{r}_t^c + K_{t+1}^c(\boldsymbol{r_t})] - f^c$ and $K_t^s(\boldsymbol{r_{t-1}}) = \mathbb{E}_t[(1+b^c)\tilde{r}_t^c + K_{t+1}^c(\boldsymbol{r_t})] - (1-\gamma^c)f^c$. It is easy to establish that $\alpha_{t-1}K_t^c(\boldsymbol{r_{t-1}}) + (1-\alpha_{t-1})K_t^s(\boldsymbol{r_{t-1}})$ equals $\pi_t(1)$ and thus the characterization of $V_t(\alpha_{t-1}, \boldsymbol{r_{t-1}})$ as given in (12) holds.

(iii) $\frac{\partial \pi_t}{\partial \alpha_t}\Big|_{0+} > 0$ and $\frac{\partial \pi_t}{\partial \alpha_t}\Big|_{1-} < 0$, i.e., $h_t^1 > 0$ and $h_t^2 < 0$. In this case, $\alpha_t^* = 1 - \alpha_{t-1}$ and $\pi_t(1-\alpha_{t-1}) = \alpha_{t-1}\left(-(1-\gamma^s)f^s + \mathbb{E}_t[(1+b^s)\tilde{r}_t^s + K_{t+1}^s(\boldsymbol{r_t})]\right) + (1-\alpha_{t-1})\left(-(1-\gamma^c)f^c + \mathbb{E}_t[(1+b^c)\tilde{r}_t^c + K_{t+1}^c(\boldsymbol{r_t}\right.$ Because $h_t^1 > 0$ and $h_t^2 < 0$, $K_t^j(\boldsymbol{r_{t-1}})$ for $j \in \{c, s\}$ as defined in (11) is such that $K_t^c(\boldsymbol{r_{t-1}}) = -(1-\gamma^s)f^s + \mathbb{E}_t[(1+b^s)\tilde{r}_t^s + K_{t+1}^s(\boldsymbol{r_t})]$ and $K_t^s(\boldsymbol{r_{t-1}}) = -(1-\gamma^c)f^c + \mathbb{E}_t[(1+b^c)\tilde{r}_t^c + K_{t+1}^c(\boldsymbol{r_t})]$. It follows that $\alpha_{t-1}K_t^c(\boldsymbol{r_{t-1}}) + (1-\alpha_{t-1})K_t^s(\boldsymbol{r_{t-1}})$ equals $\pi_t(1-\alpha_{t-1})$ and thus the characterization of $V_t(\alpha_{t-1}, \boldsymbol{r_{t-1}})$ as given in (12) holds. ■

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